Chapter 5

Aperture Antennas

Wire, loop, and patch antennas are excited by a transmission line. If the excitation is a propagating wave, instead of a transmission line, we refer to the antenna as an aperture antenna. The propagating wave is typically produced by another transducer which is itself an antenna and provides the transmission line connection for the aperture antenna. Aperture antennas include the open-ended waveguide, slotted waveguide, horn antennas, and reflectors. The feed of a parabolic dish or other type of reflector antenna is often itself an aperture antenna such as an open-ended waveguide or horn.

5.1 Antenna Efficiency and Aperture Efficiency

The fraction of the power incident on an antenna that is actually available at the terminals of the antenna is referred to as the antenna efficiency. The antenna efficiency of an aperture antenna is defined to be the ratio of the effective area to the physical aperture area,

$$\eta_{\mathrm{ant}} = \frac{A_{\mathrm{eff}}}{A_{\mathrm{p}}} \tag{5.1}$$

The aperture efficiency or aperture illumination efficiency is the ratio of the directivity to its standard directivity. The standard directivity is the maximum directivity from a planar aperture of area A_{p} when excited with a uniform amplitude aperture distribution. If the aperture dimensions are larger than the wavelength, then the standard directivity is $4\pi A_{\mathrm{p}}/\lambda^2$. If the direction of maximum directivity is steered away from broadside, then A_{p} is the projected area of the actual aperture in a direction transverse to the direction of maximum radiation intensity.

From the definition in terms of standard directivity, the aperture efficiency is

$$\eta_{\mathrm{ap}} = \frac{\lambda^2}{4\pi A_{\mathrm{p}}} D \tag{5.2}$$

Using (2.118) and $G = \eta_{\mathrm{rad}} D$ together with these efficiency definitions, we find that

$$\eta_{\mathrm{ant}} = \eta_{\mathrm{rad}} \eta_{\mathrm{ap}} \tag{5.3}$$

For a lossless antenna, antenna efficiency and aperture efficiency are equal.

5.2 Aperture Antenna Equivalent Current Model

An aperture antenna can be thought of as a hole in a conducting screen illuminated from one side by a plane wave. Aperture antennas are rarely fabricated this way, but this provides a convenient model for analysis.

The first step in analyzing the aperture antenna is to find an equivalent current model. We can accomplish this using the equivalence theorem from electromagnetic theory. The equivalence theorem states that we can replace the fields and structures on one side of a dividing surface with impressed surface currents

$$\overline{J}_s = \hat{n} \times \overline{H} \tag{5.4}$$

$$\overline{M}_s = -\hat{n} \times \overline{E} \tag{5.5}$$

We will choose the surface to align with the infinite conducting screen. These equivalent sources radiate the same fields on the aperture side and zero fields on the illumination side. If we assume that the fields are small in the plane of the aperture outside the aperture, then the equivalent sources are approximately zero except on the aperture. On the aperture, the equivalent sources are

$$\overline{J}_s = \hat{n} \times \overline{H}_a \tag{5.6}$$

$$\overline{M}_s = -\hat{n} \times \overline{E}_a \tag{5.7}$$

where $\overline{E}_a$ and $\overline{H}_a$ are the electric and magnetic fields at the aperture. At this point, we can remove the conducting screen, leaving only the equivalent sources. We then introduce an infinite ground plane on the zero field side. The ground plane shorts the electric current, since the induced current is equal in magnitude and opposite in sign, and using image theory we can remove the ground plane and double the magnetic surface current. The resulting equivalent source is then

$$\overline{M}_s = -2\hat{n} \times \overline{E}_a \tag{5.8}$$

Alternately, in the last step we could use a perfectly magnetically conducting (PMC) screen, in which case the equivalent current

$$\overline{J}_s = 2\hat{n} \times \overline{H}_a \tag{5.9}$$

is obtained.

The far field and radiation pattern for the aperture antenna can be found by applying the far field radiation integral to either of these equivalent current models. The results for the two models are similar, and only differ up the the degree of the approximation made when fringing fields near the edges of the aperture are neglected. For electrically large apertures, the fringing fields are negligible in relation to the fields over the area of the aperture, and the current models developed above are quite accurate.

5.3 Uniformly Illuminated Aperture

The simplest illumination is a constant aperture field. If the aperture is in the x-y plane and the amplitude of the electric field of the illuminating plane wave is E_0, then the far electric field in the z direction is

$$\overline{E}(\overline{r}) = -j\omega\mu(1 - \hat{r}\hat{r}\cdot)\frac{e^{-jkr}}{4\pi r}\hat{p}\frac{2E_0}{\eta}A_{\text{p}} \tag{5.10}$$

where A_{p} is the aperture area and $\hat{p}$ is a unit vector aligned with the polarization of the illuminating field. In the z direction, $\hat{r}$ and $\hat{p}$ are orthogonal, and the far field becomes

$$\overline{E}(\overline{r}) = -j\omega\mu\hat{p}\frac{e^{-jkr}}{4\pi r}\frac{2E_0}{\eta}A_{\text{p}} \tag{5.11}$$

The power density in the z direction is

$$S = \frac{A_{\text{p}}^2|E_0|^2}{2\eta\lambda^2 r^2} \tag{5.12}$$

The radiate power could be found by integrating the far field, but the integral is difficult to evaluate. Instead, we can simply integrate the power in the illuminating plane wave over the aperture. This results in

$$P_{\text{rad}} = A_{\text{p}} \frac{|E_0|^2}{2\eta} \tag{5.13}$$

With the far field power density and the radiated power, we can find the directivity of the aperture antenna,

$$D = \frac{4\pi}{\lambda^2} A_{\text{p}} \tag{5.14}$$

From (5.2), the aperture efficiency of the uniformly illuminated aperture antenna is 100%. This only holds if the aperture is electrically large enough that the equivalent current model is accurate.

In this analysis, we have only considered the directivity in the z direction, or the boresight direction of the aperture antenna. The radiation pattern can be found by evaluating the far field radiation integral for an arbitrary far field direction. The most common aperture shapes are rectangular and circular. For a rectangular aperture, the radiation pattern is given by a squared sinc function over angle, and for a circular aperture, it is a squared $J_1(x)/x$ type function, which is sometimes referred to as a jinc function, due to its similarity to the sinc function.

The uniform illumination can be modified in two ways. First, we can change the propagation direction of the illuminating plane wave to cause a phase progression across the aperture. If this is done, the main beam shifts away from the z direction to a new direction that is aligned with the direction of propagation of the incident wave. Second, we can taper the illumination so that the amplitude is nonuniform over the aperture. Most real aperture antennas have a nonuniform amplitude distribution.

The uniformly illuminated aperture has the highest directivity and the narrowest main beam of any well-behaved illumination function, but also has relatively high sidelobes. This is analogous to window theory in signal processing. When computing the power spectral density of a finite length time series, the spectral resolution achieved with a uniform window function is larger than with other window functions, but the sidelobes of the Fourier transform of the uniform window lead to spectral leakage, and other smoother windows are often used to avoid this.

5.4 Open-Ended Waveguide

While uniform illumination is easy to analyze, it is difficult to realize physically with an actual aperture antenna. The simplest practical aperture antenna is a section of waveguide with an open end. The waveguide is excited by a probe at one end, which is typically closed, and the antenna radiates from the open end. Most often, the waveguide is designed for the dominant mode operation, although multiple modes are sometimes used to control the aperture field distribution at the open end in order to shape the radiation pattern of the aperture antenna.

For the dominant TE_{10} mode, the electric field associated with the forward wave in the waveguide is

$$\overline{E} = \hat{y} E_0 \sin(\pi x/a) e^{-jk_z z} \tag{5.15}$$

At the open end of the waveguide, the wave reflects with reflection coefficient Γ. Other modes are also produced by the discontinuity in the waveguide, but we will assume that mode conversion can be neglected. This is similar to neglecting fringing fields near the edges of a parallel plate capacitor. If we place the open end of the waveguide at $z = 0$, then

$$\overline{E}(z = 0) \simeq \hat{y} E_0 (1 + \Gamma) \sin(\pi x/a) = \hat{y} E_1 \sin(\pi x/a) \tag{5.16}$$

We now want to find equivalent free-space currents at the open end of the waveguide that radiate the same fields. Using (5.8), the equivalent current model for the open ended waveguide is

$$\begin{aligned}\overline{M}_s &= -2\hat{n} \times \overline{E}_a \\ &= -2\hat{z} \times \hat{y}\sin(\pi x/a) \\ &= \hat{x}2E_1\sin(\pi x/a) \end{aligned} \tag{5.17}$$

on the aperture.

For convenience, we will shift the aperture in the x-y plane so that the center is located at the origin. The aperture current becomes

$$\overline{M}_s = 2E_1\hat{x}\cos(\pi x/a), \quad -a/2 \le x \le a/2,\ -b/2 \le y \le b/2 \tag{5.18}$$

Using the far field radiation integral, we can find that

$$\overline{E} = jk\frac{e^{-jkr}}{4\pi r}\hat{r} \times \left[\hat{x}4\pi abE_1\frac{\cos(k_x a/2)}{\pi^2-(k_x a)^2}\frac{\sin(k_y b/2)}{k_y b/2}\right] \tag{5.19}$$

where $k_x = k\sin\theta\cos\phi$ and $k_y = k\sin\theta\sin\phi$. Using $\hat{x} = \hat{r}\sin\theta\cos\phi + \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$,

$$\overline{E} = jk\frac{e^{-jkr}}{r}(\hat{\phi}\cos\theta\cos\phi + \hat{\theta}\sin\phi)abE_1\frac{\cos(k_x a/2)}{\pi^2-(k_x a)^2}\frac{\sin(k_y b/2)}{k_y b/2} \tag{5.20}$$

The two factors that determine most of the angular dependence are a sinc function and a sinc-like function. The principle pattern cuts are

$$\phi = 0, 180^\circ \qquad E_\phi \sim \cos\theta\frac{\cos(k_x a/2)}{\pi^2-(k_x a)^2} \tag{5.21a}$$

$$\phi = 90^\circ, 270^\circ \qquad E_\theta \sim \frac{\sin(k_y b/2)}{k_y b/2} \tag{5.21b}$$

For single mode operation, a and b are relatively small (on the order of a wavelength). In this case, the patterns are broad. For a larger aperture, the pattern lobes are narrower.

It is interesting to compare the two pattern cuts. Since the half-cyle sinusoidal taper is in the x direction, we expect the $\phi = 0, 180^\circ$ cut to have lower sidelobes and a broader main lobe than the $\phi = 90^\circ, 270^\circ$ cut. The numerator of (5.21a) has a zero at $k_x a = \pi$, but the denominator is also zero at that angle. Since the limit is finite, the first null of the $\phi = 0, 180^\circ$ cut does not occur until

$$k_x a = 3\pi \quad \Longrightarrow \quad \sin\theta_1 = \frac{3\pi}{ka} \tag{5.22}$$

By inspecting (5.21b), it can be seen that the first null occurs at

$$k_y b = 2\pi \quad \Longrightarrow \quad \sin\theta_2 = \frac{2\pi}{kb} \tag{5.23}$$

As an example, if the dimensions of the aperture are chosen to be $a = 3\,\lambda$ and $b = 2\,\lambda$, then the main lobe in the two pattern cuts has the same null-to-null beamwidth. The sidelobe level is larger in the $\phi = 90^\circ, 270^\circ$ because the aperture pattern is not tapered in that direction.

From the far field distribution, it is straightforward to compute the antenna gain. In computing the gain of an antenna, the total power radiated is required. This could be obtained by integrating the far field, but it is much easier to use conservation of energy to equate the total power radiated to the power passing through the aperture,

$$P_{\rm rad} = \frac{1}{2}\int_{\rm aperture} {\rm Re}[\overline{E}_a \times \overline{H}_a^*]\, d\overline{r} \tag{5.24}$$

For the rectangular aperture with dominant mode excitation, the power through the aperture is

$$P_{\rm rad} \simeq \frac{1}{2}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} \frac{1}{2\eta}|E_1|^2 \cos^2(\pi x/a)\, dx\, dy = |E_1|^2 \frac{ab}{4\eta} \tag{5.25}$$

From (5.20), the power density in the boresight direction is

$$S_r = \frac{k^2 a^2 b^2}{2\eta r^2 \pi^4}|E_1|^2 \tag{5.26}$$

From these results, the directivity is

$$D = \frac{8}{\pi^2}\frac{4\pi}{\lambda^2}ab \tag{5.27}$$

The leading factor represents the decrease in directivity due to the sinusoidal taper in the aperture field distribution.

5.5 Horn Antennas

In order to achieve high gain, we want the aperture size to be large. If we increase the size of the waveguide, we lose the advantage of single mode operation. In order to have high gain without sacrificing single mode operation, a transition from a small waveguide to a large aperture can be used. This is a horn antenna. A common type of horn is a flare from rectangular waveguide to a large rectangular aperture. The flare may be in one or both planes of the aperture. A circular waveguide can also be flared to a larger circular aperture (conical horn). Typically, the flare angle of a horn is less than 20°, since a more abrupt transition leads to strong mode conversion and degraded performance.

To analyze the radiation pattern and gain of a horn, the same expressions used for an open ended waveguide can be used. If the horn is very large, the phase of the aperture fields is no longer constant, and correction terms must be used.

5.6 Reflector Antennas

Reflector antennas can be understood using a high frequency or geometrical optics approximation. For a parabolic reflector, a spherical wave from a point source is transformed into a plane wave. Diffraction due to the finite size of the reflector causes the far field pattern to consist of a narrow distribution of plane waves, so that the radiation pattern is sinc-like and the antenna gain is large. For the reflector, the key parameters are the focal length f and the diameter D. The ratio f/D is important, since that determines the angular extent of the reflector from the point of view of the feed. Large f/D corresponds to a “shallow” reflector, and a “deep” reflector has a small f/D.

One problem with a standard paraboloidal reflector antenna is that the feed antenna at the focal point blocks some of the radiated field, leading to gain loss. An offset feed configuration can be used to reduce blockage. The reflector is an offset section of a parent paraboloid, with the feed at the focus of the parent paraboloid.

The basic tradeoff in reflector design involved the radiation pattern of the feed. If the radiation pattern is broad, some of the energy misses the reflector. If the radiation pattern is narrow, all the energy strikes the reflector surface, but the reflector is underilluminated. In both cases, the achieved gain is suboptimal. For a given aperture, maximum gain is obtained with a uniform illumination. Since the feed pattern cannot transition to zero immediately at the reflector rim, the reflector must be underilluminated to some degree. As a receiver, there is a similar tradeoff. A broad feed radiation pattern increases spillover noise from the warm ground behind the reflector. A narrow radiation pattern reduces spillover noise but decreases the amount of signal received by the feed from the reflector.

We will now examine the receiving case analytically. The two key parameters of a reflector antenna system are the aperture efficiency and the spillover efficiency. The gain of an aperture antenna

$$G = \eta_{\text{rad}}\eta_{\text{ap}}A_{\text{p}}\frac{4\pi}{\lambda^2} \tag{5.28}$$

For a reflector antenna, the aperture efficiency is typically around 60%, but can be increased to 80% by careful feed design. Gain is decreased by reflector surface abberations, so high aperture efficiency also requires a smooth and accurate reflector shape. Spillover efficiency is defined by

$$\eta_{\text{spill}} = \frac{P_{\text{illum}}}{P_{\text{feed}}} \tag{5.29}$$

where P_{feed} is the total power radiated by the feed and P_{illum} is the power incident on the reflector. In view of (2.137), (2.138) follows directly from this definition.

From (2.138), (2.140), and (5.28), the SNR at the receiver output with a conjugate matched load on the antenna terminals is

$$\text{SNR} = \frac{\eta_{\text{rad}}\eta_{\text{ap}}A_{\text{p}}S^{\text{inc}}}{k_{\text{B}}[\eta_{\text{rad}}(1-\eta_{\text{spill}})T_{\text{ground}} + \eta_{\text{rad}}T_{\text{sky}} + (1-\eta_{\text{rad}})T_{\text{p}} + T_{\text{rec}}]B} \tag{5.30}$$

where T_{rec} is the equivalent noise temperature of the receiver attached to the antenna feed port. The denominator of this expression is the equivalent system noise temperature T_{sys}. The sensitivity or figure of merit (2.143) is

$$\frac{G}{T_{\text{sys}}} = \frac{G}{\eta_{\text{rad}}(1-\eta_{\text{spill}})T_{\text{ground}} + \eta_{\text{rad}}T_{\text{sky}} + (1-\eta_{\text{rad}})T_{\text{p}} + T_{\text{rec}}} \tag{5.31}$$

The sensitivity can also be expressed as

$$\frac{A_{\text{eff}}}{T_{\text{sys}}} = \frac{\eta_{\text{rad}}\eta_{\text{ap}}A_{\text{p}}}{\eta_{\text{rad}}(1-\eta_{\text{spill}})T_{\text{ground}} + \eta_{\text{rad}}T_{\text{sky}} + (1-\eta_{\text{rad}})T_{\text{p}} + T_{\text{rec}}} \tag{5.32}$$

in units of m^2/K.

For a small feed with broad illumination pattern, both the aperture efficiency and the spillover efficiency are small, leading to poor SNR. For a very large feed with a narrow illumination pattern, spillover efficiency is very close to one and the spillover noise is small, but the reflector surface is underilluminated and the aperture efficiency is poor. Maximum SNR is obtained with an illumination pattern that is a compromise between these two extremes. In general, the optimal solution depends on the receiver noise temperature, but if the receiver noise temperature is large, spillover noise is unimportant and the feed should be designed for maximum gain. A standard compromise is an illumination pattern that tapers to -10 dB at the reflector rim.

The spillover efficiency of a reflector antenna system can be found by integrating the feed radiation pattern according to (2.137) or (5.29). The gain and aperture efficiency are found by computing the far field radiated by an equivalent current distribution on the reflector surface. Typically, the equivalent current is obtained from the physical optics (PO) approximation. When an incident wave strikes a flat PEC plane, the induced current is $\overline{J}_s = 2\hat{n} \times \overline{H}^{\text{inc}}$. For a smooth conductor without sharp corners, the curvature causes a deviation of the surface current from this expression, but as long as the conducting surface is nearly flat on the scale of a few wavelengths, the current can be approximated from the expression for a flat surface as

$$\overline{J}_s \simeq 2\hat{n} \times \overline{H}^{\mathrm{inc}} \tag{5.33}$$

The far fields can be obtained by treating $\overline{J}_s$ as an impressed current in free space and using the radiation integral. For very large reflectors, evaluating the radiation integral can take a long time. There are a number of fast ways to do this, including the use of the FFT and various types of expansions of the current.

For a large reflector, physical optics is quite accurate, especially near the main lobe of the radiation pattern. A correction term can be added for a line current inducted on the reflector rim, which takes into account edge diffraction. The combination of PO and edge diffraction using a line current is known as the physical theory of diffraction (PTD).