2.2 Antenna Parameters

An antenna is a transformer between a transmission line and free space. The antenna has a waveguide port that represents its input and output terminals, and a radiating element that couples to waves in space. When operating as a transmitter, the antenna accepts a wave propagating as a mode along a waveguide or transmission line, and emits a radiating wave into free space. As a receiver, the antenna intercepts a wave in free space and introduces a propagating wave on a transmission line.

To describe an antenna, we must characterize its properties as a transmission line load (input impedance) and the distribution of the electromagnetic energy that it radiates into space (radiation pattern). There are a number of key parameters and concepts that we can use to describe antenna properties, such as the gain, which characterizes the directionality of the fields as they radiate away from the antenna, and the radiation resistance, which represents the portion of the power input to the antenna port that radiates into space instead of being absorbed by the antenna through ohmic losses or reflected back down the feeding transmission line. We will first consider antennas as transmitters, and then we will use the electromagnetic reciprocity principle to determine the receiving properties of an antenna.

2.2.1 Antenna Parameter Definitions

Many antenna parameters are defined in terms of the electromagnetic field radiated by the antenna. The first step in antenna analysis is to find the radiated fields, which can be done using an analytical approximation for the current distribution on the antenna together with the radiation integral discussed in the previous section or a numerical method implemented in software. Once the radiated fields are known, the antenna radiation pattern can be obtained and various properties of the pattern, such as gain and beam width, can be quantified.

The source document for the definitions of antenna parameters is the IEEE Standard Definitions of Terms for Antennas [2]. In antenna textbooks and papers, authors generally agree on the basic terms, but sometimes different meanings are assigned to parameters like aperture efficiency and antenna efficiency. In this book, we will always be consistent with the IEEE standard.

Some of the most commonly used terms are described below.

Radiation pattern: An antenna radiation pattern is the angular distribution of the power radiated by an antenna. If an antenna radiates fields $\overline{E}(r,\theta,\phi)$ and $\overline{H}(r,\theta,\phi)$, then the time average far field power density radiated at the point $\overline{r}$ has the form

$$\begin{aligned}\overline{S}_{\rm av}(r,\theta,\phi) &= \tfrac{1}{2}\mathrm{Re}\left[\overline{E}(\overline{r})\times\overline{H}(\overline{r})^*\right] \\ &= \frac{|\overline{E}(\overline{r})|^2}{2\eta}\hat{r} \\ &\simeq f(\theta,\phi)\frac{1}{r^2}\hat{r}, \quad r\to\infty \end{aligned} \tag{2.80}$$

where we have lumped all the angle dependence of the power density into $f(\theta,\phi)$. The steps in this derivation follow from the fact that $\overline{E}$ and $\overline{H}$ are mutually orthogonal in the far field and lie in the θ,ϕ plane, and the amplitude of $\overline{H}$ is related to the amplitude of $\overline{E}$ by a factor of $1/\eta$. The angular dependence of the radiated power density is the radiation pattern of the antenna. It is customary to normalize the radiation pattern to a maximum value of unity, so that the radiation pattern is defined to be $f(\theta,\phi)/f_{\max}$, where $f_{\max}$ is the maximum value of $f(\theta,\phi)$. While the power density pattern is most important, for some applications it is important to consider also the angular distributions of the phase or polarization properties of the radiated far fields.

Isotropic pattern: Equal power is radiated in all directions, so that $f(\theta, \phi) = 1$. No real antenna has an isotropic radiation pattern, but the isotropic radiator is important as a reference pattern with which to compare other antenna radiation patterns. For a given antenna, the larger the peak value of the radiation pattern relative to the radiation pattern of an idealized isotropic radiator, the larger the directivity of the antenna. For an isotropic radiator, the radiated power is spread equally over a sphere with area $4\pi r^2$, where r is the distance from the antenna. The power flux density in all directions is $P_{\mathrm{rad}}/(4\pi r^2)$, where P_{rad} is the total radiated power.

Omnidirectional pattern: The radiation pattern is of the form $f(\theta, \phi) = f(\theta)$, so the pattern depends on elevation but is independent of the azimuthal angle ϕ.

Pattern cut: In general, a radiation pattern is a two-dimensional function of θ and ϕ, which requires a 3D plot to visualize or represent graphically. For convenience, it is common to plot a slice of the pattern, such as a theta cut, $f(\theta, \phi_0)$ with ϕ_0 fixed.

E-plane cut: Pattern in the plane containing the electric field vector radiated by the antenna and the direction of maximum radiation.

H-plane cut: Pattern in the plane containing the magnetic field vector radiated by the antenna and the direction of maximum radiation. The E-plane and H-plane cuts are orthogonal.

Pattern lobes: Local maxima in the radiation pattern. The main lobe is the lobe for which the radiation pattern reaches its peak value. Sidelobes are smaller lobes, which typically decrease in amplitude in angle away from the main lobe. A back lobe is a pattern lobe near the opposite direction of the main lobe.

Sidelobe level: Ratio of $f_{\max}$ to the peak value of the largest sidelobe. The sidelobe level is commonly expressed as a positive value in dB, so that the sidelobe level represents how far the largest sidelobe is below the main lobe peak.

Beamwidth: There are several ways to specify the angular width of the main lobe. The most common are the half-power beamwidth (HPBW) and null to null beamwidth. HPBW is the angular distance in a pattern cut between the points where the radiation pattern is half the value at the peak of the main lobe. Null to null beamwidth is the angle between the two pattern nulls adjacent to the main lobe. The half-power half beamwidth (HPBW/2) is also used occasionally.

Pencil beam pattern: Radiation pattern with a small main beamwidth. Most of the radiated power is directed in a small angular range, rather than spread evenly over a wide range of angles. This type of antenna has a high directivity.

Directivity: Ratio of radiated power density in a given direction to the power density that would be radiated by an isotropic reference antenna with the same total power. In terms of the time-average radiated power density $\overline{S}_{\mathrm{av}}$, the total radiated power is

$$P_{\mathrm{rad}} = \oint_S \overline{S}_{\mathrm{av}} \cdot d\overline{S} \tag{2.81}$$

where S is a closed surface containing the antenna. The power density radiated by an isotropic antenna is

$$\overline{S}_{\mathrm{iso}} = \hat{r}\frac{P_{\mathrm{rad}}}{4\pi r^2} \tag{2.82}$$

By the definition, the directivity pattern is

$$D(\theta, \phi) = \frac{S_{\rm av}(\overline{r})}{P_{\rm rad}/(4\pi r^2)} \tag{2.83}$$

The directivity of the antenna is the maximum value of the directivity pattern. Directivity and beamwidth are inversely related, since a high directivity means that the power density is much higher in one direction than in other directions, which necessarily corresponds to a narrow main beam.

Polarization: The polarization of an antenna in a given direction is the polarization of the wave transmitted by the antenna in that direction. The polarization can be linear, circular, or elliptical. A dual-polarization antenna is an antenna with two ports, one that induces a transmitted wave with a given polarization, and a second port that induces a wave with a polarization that is nominally orthogonal to the polarization of the first wave.

Partial directivity: In many cases, a receiver does not capture all the power radiated by the transmitting antenna, but instead only the power in one polarization. The polarization can be defined by a unit vector $\hat{p}$. Partial directivity is defined to be the radiation intensity corresponding to a given polarization divided by the total radiated power averaged over all directions [2]. We can modify (2.83) to be the partial directivity with respect to the polarization $\hat{p}$ by replacing the radiated power density with the radiated power density in one polarization,

$$S_{\rm av,p} = \frac{|\hat{p}\cdot\overline{E}(\overline{r})|^2}{2\eta} \tag{2.84}$$

in the definition (2.83) of directivity.

Radiation efficiency: Ratio of the total radiated power to the input power or the power $P_{\rm in}$ accepted by the antenna at its input terminals,

$$\eta_{\rm rad} = \frac{P_{\rm rad}}{P_{\rm in}} \tag{2.85}$$

For an ideal, lossless antenna, $\eta_{\rm rad} = 1$.

Radiation resistance: Real part of the input impedance of a lossless antenna as a transmission line load due to energy radiated from the antenna into space. The portion of the real resistance due to radiation can be found from the total radiated power and the input current to the antenna using

$$R_{\rm rad} = \mathrm{Re}[Z_{\rm in}] = \frac{P_{\rm rad}}{\frac{1}{2}|I_{\rm in}|^2} \tag{2.86}$$

where $I_{\rm in}$ is the input current that was assumed when approximating or computing the radiated fields. A lossy antenna has an additional contribution in its input resistance due to ohmic resistance in the antenna structure or dielectric losses.

Gain: Gain is similar to directivity, but with the input power in the denominator instead of the radiated power. The gain pattern is

$$G(\theta, \phi) = \frac{S_{\rm av}(\overline{r})}{P_{\rm in}/(4\pi r^2)} \tag{2.87}$$

Since the input power is always greater than the radiated power for a passive antenna, gain is less than or equal to directivity. The difference between the input power and the radiated power is the power absorbed in the antenna by dielectric or conductor losses. Gain and directivity are related by the radiation efficiency, so that $G(\theta, \phi) = \eta_{\rm rad} D(\theta, \phi)$. Directivity assumes a lossless antenna and ignores power reflected at the input port and absorbed in the antenna, whereas gain includes the effect of losses in reducing the power radiated by the antenna. The gain G of an antenna is the maximum value of the gain pattern $G(\theta, \phi)$.

Other efficiencies: There are several other efficiencies which are used as figures of merit for various types of antennas. These include polarization efficiency, which is discussed below, and reflection or matching efficiency, which is the ratio of power accepted by an antenna to the power that would be supplied by the generator to a matched line and is less than one if the reflection coefficient looking into the antenna input port is not one. Realized gain is gain multiplied by the reflection efficiency. Aperture antennas have an aperture efficiency. Antenna efficiency is similar to aperture efficiency but includes antenna losses, so that aperture efficiency is to antenna efficiency as directivity is to gain. Many authors define efficiencies in various nonstandard and conflicting ways, but the governing conventions that should be used generally in antenna work are given by the IEEE Standard Definitions of Terms for Antennas [2].

2.2.2 Antenna Physical Size and Frequency

Many antennas are operated at resonance, which means that at the desired operating frequency, the antenna input impedance is purely real. At higher or lower frequencies, the antenna input impedance becomes reactive. A dipole antenna, for example, is resonant when its length is just under a half wavelength at the given operating frequency. At higher frequencies, the antenna is inductive, and at lower frequencies it is capacitative. In general, there is a rough correspondence between operating wavelength and antenna size, meaning that antennas for lower frequencies are physically large, and antennas for high frequencies can be smaller.

The electrical size of an antenna also has an impact on the radiation pattern and radiation efficiency. For an electrically small antenna ($d \ll \lambda$), the pattern is broad and the radiation resistance small. If the radiation resistance is small, then a large input current must be driven at the antenna input port to radiate an appreciable amount of power, which leads to large ohmic losses in the antenna and consequently a low radiation efficiency. A large antenna ($d \gg \lambda$) tends to have a narrow main lobe and high gain. A resonant antenna ($d \simeq \lambda$) typically has good radiation efficiency only in a narrow band.

For most applications, it is desirable to have as small an antenna as possible while meeting the desired gain, efficiency, and bandwidth requirements. We have just pointed out that small antennas are often inefficient due to low radiation resistance in relation to the ohmic resistance of the antenna. Even if the antenna is low loss, an electrically small antenna still has fundamental performance limitations on its bandwidth. Because a small antenna has a reactive input impedance, a matching circuit is required in order to couple power into or out of the antenna. If antenna losses are small, the smaller the antenna, the higher the quality factor of this matching circuit and the narrower the bandwidth. Therefore, it is difficult to achieve high efficiency and broad bandwidth with an antenna that is very small in relation to the operating wavelength. This issue is dealt with in more detail in Section 3.4.

While antenna analysis often focuses on far field properties of the radiated fields, the behavior of fields close to the antenna are also important. The reactive near-field is a small region near the antenna where fields are dominated by $1/r^3$ terms which represent stored energy. The Fresnel zone is farther out, with fields that are predominantly radiative in nature rather than reactive, but the angular distribution of the radiated field intensity around the antenna still depends on distance from the antenna. For $r > 2d^2/\lambda$, where d is the size of the antenna, the far field approximation is accurate, and the angular dependence of the fields can be separated from the radial dependence as in (2.78).

2.2.3 Antenna Polarization

At a given location far from a transmitting antenna, the spherical wave radiated by the antenna can be approximated as a plane wave. A electromagnetic plane wave is a simple solution to Maxwell's equation in a source-free region for which the phase fronts of the wave are planar. An plane wave can be described by its frequency, direction of propagation, amplitude, and polarization. The direction of propagation is in the radial direction $\hat{r}$, which always points away from the antenna, and the amplitude can be found from the radiation integral formulation given above. The remaining degree of freedom for the field radiated by an antenna is the polarization.

Polarization describes the directions of the electric and magnetic fields associated with the wave. Since the electric and magnetic fields are perpendicular for a plane wave in an isotropic medium, it suffices to know only one of the directions. By convention, we specify the polarization of a plane wave using the electric field. Rigorously speaking, the polarization of a wave is the figure traced by the tip of the electric field vector in time, viewed in the direction of propagation.

The three possible wave polarizations are as follows:

Linear: If the phasor electric field for a plane wave is $\overline{E}(\overline{r}) = \hat{x}E_0e^{-jkz}$, the polarization can be found by computing the time-varying electric field,

$$\begin{aligned}\overline{E}(\overline{r},t) &= \text{Re}\left[\hat{x}E_0e^{-jkz}\right]\\ &= \hat{x}|E_0|\cos(\omega t - kz + \phi)\end{aligned}$$

where ϕ is the phase angle of the constant E_0. As time increases, the electric field vector moves along the x axis on the line segment between $|E_0|$ and $-|E_0|$. Because the figure traced by the electric field vector is a line, we refer to the polarization is linear. If the electric field has both $\hat{x}$ and $\hat{y}$ components that are in phase, then the polarization is also linear, but rotated at an angle relative to the x axis that depends on the magnitudes of the x and y components.

Circular: If $\overline{E}(\overline{r}) = (\hat{x} + j\hat{y})e^{-jkz}$, the time-varying electric field is

$$\begin{aligned}\overline{E}(\overline{r},t) &= \text{Re}\left[(\hat{x} + j\hat{y})E_0e^{-jkz}\right]\\ &= \hat{x}\cos(\omega t - kz) - \hat{y}\sin(\omega t - kz)\end{aligned}$$

This vector traces out a circle in the xy-plane. The sense of the rotation is left-handed with the thumb in the direction of propagation, so the polarization is left-hand circular (LHCP). If the sign of the $\hat{y}$ component of the field is reversed, then the polarization is RHCP.

Eliptical: If $\overline{E}(\overline{r}) = (E_x\hat{x} + E_y\hat{y})e^{-jkz}$ with $|E_x| \neq |E_y|$ and E_x and E_y are out of phase, then the electric field vector traces an ellipse. Elliptical is the most general form of the polarization, and linear and circular can be viewed as degenerate and special cases of elliptical, respectively.

The polarization of an antenna in a given direction is defined to be the polarization of the fields radiated by the antenna in that direction. For a high gain antenna, the polarization in the direction of the main beam is most important, but in some applications, the polarization pattern over angle is also relevant. Most antennas transmit or receive linearly polarized fields, but antennas can also be designed for circular polarization or dual polarization.

Polarization Efficiency

If the polarization of a receive antenna is not the same as the polarization of an incoming wave, less power than the maximum available power in the wave is received. This reduction in power received can be quantified using polarization efficiency. If the incident field on a receiving antenna is

$$\overline{E}^i = \hat{p}_i E_0 e^{-j\overline{k}^i\cdot\overline{r}}$$

and the polarization of the receive antenna is $\hat{p}_a$, then the polarization efficiency is

$$\eta_{\text{pol}} = |\hat{p}_i \cdot \hat{p}_a|^2 = |\cos(\psi_{\text{pol}})|^2 \tag{2.88}$$

where ψ_{pol} is the angle between the polarizations of the incident field and the antenna. If the two polarizations are orthogonal, no power is received and $\eta_{\text{pol}} = 0$. This expression holds for linear, circular, and elliptically polarized incident waves and receiving antennas.

2.3 Hertzian Dipole

The simplest model for an antenna is the Hertzian dipole. A dipole antenna is a length of wire with a small feed gap driven by a current source. The current is necessarily zero at the ends of the wires, and the current strength changes roughly sinusoidally along the wire. For a very short dipole, the current behaves like a small part of a sinusoid near where the sine function is linear, so that the current changes almost linear from the maximum near the feed gap to zero at the ends of the wires, symmetrically in both directions away from the feed gap.

To a first approximation we can ignore the variation of the current along the wires and assume a constant current distribution. A constant current distribution on a segment of length l on the z axis is represented by the current density

$$\overline{J}(\overline{r}) = \hat{z}\delta(x)\delta(y)I_0, \quad |z| \le l/2 \tag{2.89}$$

The delta functions in x and y confine the current to an infinitesimally thin filament along the z axis, and the current is nonzero over an interval in the z direction of length l. This is a rough model for the current on a pair of short wires excited by a source or transmission line at a small gap located at $z = 0$ in between the two wires.

If the length l of the dipole is short in relation to the wavelength (so that $l \ll \lambda$), then the exponent of the phase term in the vector current moment integration is close to zero over the range of integration. In this case, we can neglect the effect of the finite length of the dipole in the z direction on the integration, and the equivalent current can be written as

$$\overline{J}(\overline{r}) = \hat{z}I_0 l\delta(x)\delta(y)\delta(z) = \hat{z}I_0 l\delta(\overline{r}) \tag{2.90}$$

where I_0 is the total phasor current into the feed gap, l is the total length of the antenna, and the antenna is oriented along the z axis. The notation $\delta(\overline{r} - \overline{r}_0)$ is a shorthand notation for the three-dimensional delta function $\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$. The delta function $\delta(\overline{r})$ without any offset in each of the delta functions (i.e., $\overline{r}_0 = 0$) means that the antenna is located at the origin. The current source (2.90) is essentially identical to the point source in (2.40).

To analyze the properties of the Hertzian dipole antenna model, the first step is to find the current radiated by the antenna. Using the far field radiation integral, the electric field radiated by the Hertzian dipole is

$$\overline{E}(\overline{r}) \simeq j\omega\mu \frac{e^{-jkr}}{4\pi r}\hat{\theta} I_0 l \sin\theta \tag{2.91}$$

The radiated power flux density is

$$\overline{S}(\overline{r}) \simeq \frac{\eta}{2}\left(\frac{kI_0 l}{4\pi r}\right)^2 \sin^2\theta\,\hat{r} \tag{2.92}$$

from which it can be seen that the radiation pattern is $f(\theta) = \sin^2\theta$. Integrating the power flux density over a sphere around the antenna shows that

$$P_{\text{rad}} = \eta\frac{(kI_0 l)^2}{12\pi} \tag{2.93}$$

The directivity pattern is

$$D(\theta) = \frac{3}{2}\sin^2\theta \tag{2.94}$$

and the directivity is $3/2$.

The radiation resistance is

$$R_{\text{rad}} = \eta\frac{(kl)^2}{6\pi} \tag{2.95}$$

This result is only accurate for small values of the electrical dipole length kl. As kl increases, the variation of the current along the wire begins to be significant, and a more accurate model for the current on the wire than (2.90) is required. This will be considered in Chapter 3. After treating the simple Hertzian dipole model for a small antenna, it would be natural to move on to the analysis of antennas with more complex current distributions. Before doing that, we will develop some results that apply to receiving antennas.